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Abstract

This paper addresses the problem of supply chain design at the strategic level when production/distribution of a new market opportunity has to be launched among a set of qualified partners. The new market opportunity is characterized by a deterministic forecast over a planning horizon. The product (or service) is assumed to be produced (or provided) in a fixed sequence of some distinct stages or echelons, and each stage could have a number of qualified partners. The qualified partners at each stage might be different factories (or service organization) of the same corporation or different organizations that have agreed to participate in an open supply web. We assume that the partners under consideration are equally capable at their respective processing stage (possibly through a preprocessing evaluation for the new opportunity), but the partners may differ in available capacities and costs, given their current workloads. The objective is to design the supply chain by selecting one partner from each stage to meet the forecasted demand without backlog and optimize fixed alliance related and variable production and logistics costs over the given planning horizon. A solution algorithm is developed based on key properties of the problem, and is tested on empirical data sets. The overall contribution is an analytical tool that can be employed by the coordinator of the new market opportunity at the strategic level for designing supply chains based on integrated cost concerns.

Keywords : Supply Chain Design, Strategic Decisions, Qualified Partners.

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Conception d'une chaîne d'approvisionnement avec un ensemble de partenaires pour faire face à une opportunité du marché

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Ce papier concerne le problème de la conception de chaînes d'approvisionnement au niveau stratégique quand une nouvelle opportunité du marché doit être saisie en collaboration avec des partenaires qualifiés. L'opportunité qui se présente est caractérisée par une prévision de consommation sur un horizon donné. Le produit (ou service) est supposé être fabriqué (ou mis à disposition) après plusieurs étapes (encore appelées échelons) distinctes, et à chaque étape, plusieurs partenaires qualifiés peuvent être disponibles. Les partenaires qualifiés disponibles à chaque étape peuvent être des sociétés différentes ou des éléments d'une même organisation qui ont donné leur accord pour participer au réseau qui va collaborer. Nous considérons que les partenaires situés à une même étape sont également compétents mais peuvent avoir des capacités et des coûts différents du fait de leur charge actuelle. L'objectif est de concevoir la chaîne d'approvisionnement en choisissant un partenaire à chaque étape de façon à satisfaire la demande prévisionnelle sans retard et moindre coût sur l'horizon donné. Nous proposons un algorithme et nous le testons sur des numériques. La contribution de ce papier est un outil d'aide à la stratégie des chaînes d'approvisionnement.

Mots-clefs : Conception de chaîne d'approvisionnement, décision stratégiques, partenaires qualifiés

1 Introduction

Recent manufacturing paradigms like agile manufacturing, virtual enterprises, and supply webs have resulted in interesting possibilities for companies to collaborate with each other by forming opportunistic alliances. Alongside, advances in supply chain management have elevated our consciousness to the needs and provided methods for the efficient operation and coordination of constituent elements of a typical supply chain.

In either case it is well recognized today that the success of individual companies is tied in major part to their ability to form virtual organizations rapidly to meet an emerging time-based opportunity. Due to the speed with which virtual organizations need to be formed, prequalification agreements between companies usually enhances their ability to work efficiently once the opportunity emerges. Such a collection of companies has been termed as an organizational web. More precisely, according to (Goldman, Nagel, and Preiss 1995) an organizational web is an open-ended collection of prequalified partners that agree to form a pool of potential members of virtual organizations.

Due the focus on core competency companies have become specialized and the value of the web lies in the ability of the members to form opportunistic alliances to compete for contracts that they could not win individually. Thus, the strategic problem faced by the web once a market opportunity emerges is that a unique combination of companies is to be drawn from this pool based on available capacities or capabilities that serve distinctive requirements of the customer.

We call this problem as a strategic supply chain design problem for a new market opportunity, from a set of qualified partners. Of course the qualified partners could include multiple and geographically distributed production facilities owned by the same parent company. Let us assume that the new market opportunity is characterized by a deterministic forecast over a given planning horizon. And the product is assumed to be processed in a fixed sequence of some distinct stages or echelons, and each stage could have a number of qualified partners. Many industries such as semiconductor (silicon ingot to wafer to masked wafers to chips, and so on to final integrated circuit delivery) and automotive (with tier structure of suppliers) fit this description.

The objective of this strategic problem is to design the supply chain by selecting one partner from each stage (or echelon) to meet the forecasted demand without backlog. Now it may not be necessary that only one partner is chosen for each echelon in practice. We use this model for the fact that there are additional coordination costs involved in setting up and coordinating multiple suppliers. Also in a reliable situation with adequate capacity, it can be shown that the network would be optimized and reduced to a chain. When designing the chain and delivery without backlog, it would make most sense to optimize the wholistic cost of the market opportunity. This cost could be broken down into fixed costs related to alliance/interface between adjacent members in the chain, and variable production, holding and logistics/shipment costs. Once again for simplicity we assume that the forecasted demand is not lumpy and production and delivery is expected for all periods of the planning horizon. The problem can be considered as a restricted version of the general problem considered by (Sarmiento 2001).

The remainder of the paper is organized as follows. Section 2 presents the review of literature related to the problem. Section 3 presents the problem description and formulation. The solution approach based on key properties is presented in Section 4. Numerical results of a computational study are presented in Section 5. Finally, conclusions are presented in Section 6.

2 Literature Review

The performance of supply chains, in terms of operational cost, is of great importance in the decision making process at the organizational web level. Integrated analyses, however, are very complex due to the fact that several functions are simultaneously considered into a single optimization model. In recent years, these types of problems have attracted the attention of researchers and some models have been proposed in this direction. The basic idea behind these models is to simultaneously optimize decision variables of different functions that have traditionally been optimized sequentially, in the sense that the optimized output of one stage becomes the input to the other (first setting inventory levels and then scheduling distribution, for instance). This approach has proven to be of significant relevance to companies that have adopted it. Most of the integrated analyses, however, have focused on systems composed by two echelons of the supply chain due to complexity, see for instance (Ernst and Pyke 1993), (Chandra and Fisher 1994), (Ishii, Takahashi, and Muramatsu 1988), (Blumenfeld, Burns, Diltz, and Daganzo), etc. For a review on these models the reader is referred to (Sarmiento and Nagi 1999) and (Thomas and Griffin 1996).

The problem considered in this work concerns the design of a supply chain and is strategic in nature. The strategic planning level refers to decisions taken for the long term operation of the enterprise; at this level, the primary interest is to develop an integrated, coordinated and consistent long-term plan of action. Following is a review of papers that have analyzed the entire supply chain at a strategic level.

In (Cohen and Lee 1988) a strategic model structure and a hierarchical decomposition approach for the supply chain are presented. The scope of the work presented is to analyze interactions between functions in a complete supply chain network. To model these interactions the authors consider four submodules where each represents a part of the overall supply chain: (1) material control, (2) production control, (3) finished goods stockpile, and (4) distribution network control. Stochastic considerations are incorporated in the submodules and relevant costs for set-up, inventory holding and shortage are considered. The purpose of the framework is to predict the impact on performance of alternative manufacturing strategies, and to develop an analytically based methodology to answer the following questions: (1) how can production and distribution control policies be coordinated to achieve synergies in performance, and (2) how do service level requirements for material input, work-in-process and finished goods availability affect costs, lead times and flexibility? This framework represents an important piece in the analysis of interactions in the supply chain, since it accounts for the linkages in performance measures between the four functions considered.

A model that supports resource deployment decisions in a global manufacturing and distribution network is developed in (Cohen and Lee 1989). The network considers raw material suppliers, manufacturing plants, distribution channels, warehousing locations and customers' geographical dispersion. The problem is formulated as a mixed integer, non-linear program whose objective is to maximize after-tax profits in all countries in which the firm operates. Costs considered are: variable and fixed for procurement, production, distribution and transportation as well as tariffs, duties and transfer pricing. The model is a useful tool for the evaluation of global manufacturing strategy alternatives. For an extensive review on strategic production-distribution models in a global supply environment, the reader is referred to (Vidal and Goetschalckx 1997) and (Govil and Proth 2002).

The dynamics of an integrated supply chain using the Forrester production distribution system as a reference model is analyzed in (Wikner, Towill, and Naim 1991). The authors discuss the integration of the chain and present five approaches that could improve its dynamics. They conclude that a better use of information flow in the chain can significantly improve the operational performance.

Our work differs from that in (Cohen and Lee 1989) in that the authors analyze a global supply chain, considering factors that concern international manufacturing strategies, such as tax rates, currency exchange, etc. Also, their model is used to determine resource deployment, given a logistics structure, whereas ours aims at determining the supply chain from a set of alternative options. In our approach, we assume that the partners that may participate in the new project have already agreed to collaborate, and their load/remaining capacity is known over a given horizon. We also assume that the demands are known over the same horizon. Thus, our goal is to show how to orchestrate an optimal solution for a new project through a supply chain design.

3 Problem Description and Formulation

3.1 Description

We consider virtual enterprises that manufacture products and take into consideration the following costs: unit production and holding cost for each member of the virtual organization, and transportation and a fixed cost between two successive members in the chain. This fixed cost represents all the managerial costs involved in the collaborative operation of the successive members, costs of setting-up data communication systems and finance and managerial activities, for instance. These costs may vary from echelon to echelon and from partner to partner (and possibly from period to period).

We assume that the coordinator of the organizational web has information about the sequence of operations (one operation in each echelon) that have to be performed on the product to be produced by the virtual enterprise. This sequence of operations is defined as $P =< 1, 2, ..., \varphi >$.

The problem is represented as a network $G = (N, A)$ where N, the set of nodes, represents manufacturing locations in the organizational web, and A, the set of arcs, refers to the possible connections between some pairs of nodes. A subset $E_\alpha \subseteq N$ such that all nodes in E_α are manufacturing or service locations capable of performing operation $\alpha, \alpha = 1, ..., \varphi$, is defined as an echelon of G. The set of arcs A is such that no arc exists between any pair of nodes in E_α $\forall\ \alpha \in P$, and there exists directed arc(s) from a node in E_α to one (or more) node(s) in $E_{\alpha+1}, \alpha = 1, 2, ..., \varphi - 1$. In order to design the supply chain from the organizational web, network G should contain a number of echelons equal to φ, $E_\alpha \neq \emptyset$ $\forall\ \alpha \in P$ and there should exist at least one chain connecting a node in E_1 to a node in E_φ. Figure 1 shows a network with 4 echelons.

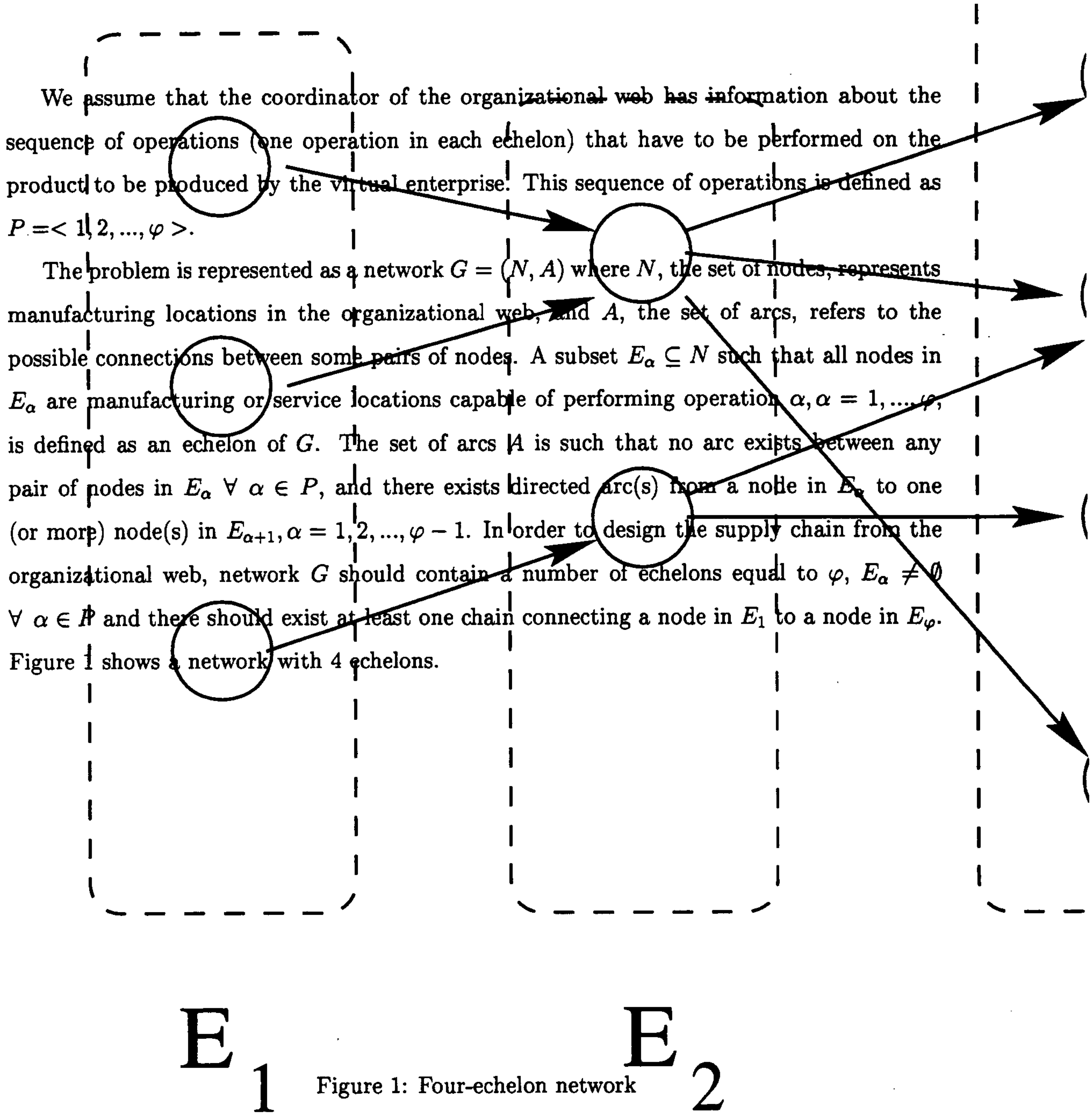

Figure 1: Four-echelon network

The problem can be viewed as a network design with fixed charge components. It is a fixed-charge problem in which production will occur only at the nodes in the chain selected. The following assumptions are made:

1. Single market opportunity (product and/or service).
2. Demand is deterministic but not constant over the planning horizon, and is fulfilled only by nodes in the final echelon of the network.

3. Backorders are not permitted at the last echelon.
4. There are capacity constraints at every node and the associated capacities may differ from node to node within an echelon and from echelon to echelon.
5. Operation time for the total units produced in a period plus transportation time to the next node in sequence is one period.
6. The transportation system is uncapacitated.

3.2 Mixed Integer Linear Programming Formulation

Parameters given:

P = $< 1, ..., \varphi >$, sequence of operations (echelons) required to manufacture the product,

N_α = number of nodes (manufacturing or service locations) in echelon α,

$F_{i,\alpha,j,\alpha+1}$ = fixed set-up cost making the connection between node i in echelon α to node j in echelon $\alpha + 1$,

$C_{i,\alpha,j,\alpha+1,t}$ = transportation unit cost from node i in echelon α to node j in echelon $\alpha + 1$ in period t,

$H^f_{i,\alpha,t}$ = holding cost at the exit of node i in echelon α in period t,

$H^r_{i,\alpha,t}$ = holding cost at the entrance of node i in echelon α in period t,

$U_{i,\alpha,t}$ = unit processing cost at node i in echelon α in period t,

T = last period in the planning horizon when a demand exists,

d_t = forecasted demand in period t,

M = a very large number, greater than the sum of the forecasted demand over all periods, and

$\Phi_{i,\alpha,t}$ = production capacity available for the new project of node i in echelon α in period t (number of units).

Variables:

$$h^f_{i,\alpha,t} \;=\; \text{amount of finished goods at node } i \text{ in echelon } \alpha \text{ in period } t,$$

$$h^r_{i,\alpha,t} \;=\; \text{amount of raw material at node } i \text{ in echelon } \alpha \text{ in period } t,$$

$$z_{i,\alpha,t} \;=\; \text{amount produced at node } i \text{ in echelon } \alpha \text{ in period } t,$$

$$x_{i,\alpha,j,\alpha+1,t} \;=\; \text{amount of product shipped from node } i \text{ in echelon } \alpha \text{ to node } j \text{ in echelon } \alpha+1 \text{ in period } t,$$

$$W_{i,\alpha} \;=\; \begin{cases} 1 & \text{if node } i \text{ in echelon } \alpha \text{ is included in the chain} \\ 0 & \text{otherwise,} \end{cases}$$

$$Y_{i,\alpha,j,\alpha+1} \;=\; \begin{cases} 1 & \text{if node } i \text{ in echelon } \alpha \text{ and node } j \text{ in echelon } \alpha{+}1 \text{ are included in the chain} \\ 0 & \text{otherwise.} \end{cases}$$

The objective function of the mixed-integer linear program is the minimization of the integrated costs for production, inventory holding, transportation and company-selection.

$$(P) \quad Min \left\{ \sum_{\alpha=1}^{\varphi-1} \sum_{j=1}^{N_{\alpha+1}} \sum_{i=1}^{N_\alpha} F_{i,\alpha,j,\alpha+1} Y_{i,\alpha,j,\alpha+1} + \sum_{\alpha=1}^{\varphi-1} \sum_{t=\alpha}^{T-\varphi+\alpha-1} \sum_{j=1}^{N_{\alpha+1}} \sum_{i=1}^{N_\alpha} C_{i,\alpha,j,\alpha+1,t} x_{i,\alpha,j,\alpha+1,t} + \sum_{\alpha=1}^{\varphi} \sum_{t=\alpha}^{T-\varphi+\alpha} \sum_{i=1}^{N_\alpha} (H^f_{i,\alpha,t} h^f_{i,\alpha,t} + H^r_{i,\alpha,t} h^r_{i,\alpha,t} + U_{i,\alpha,t} z_{i,\alpha,t}) \right\}$$

subject to:

$$\sum_{i=1}^{N_\alpha} W_{i,\alpha} = 1 \qquad \alpha = 1,\ldots,\varphi, \tag{1}$$

$$Y_{i,\alpha,j,\alpha+1} \geq W_{i,\alpha} + W_{j,\alpha+1} - 1 \qquad \forall\ (i,j) \in A; \qquad \alpha = 1,\ldots,\varphi-1, \tag{2}$$

$$z_{i,\alpha,t} \leq \Phi_{i,\alpha,t} W_{i,\alpha} \qquad i = 1,\ldots,N_\alpha; \quad \alpha = 1,\ldots,\varphi; \quad t = \alpha,\ldots,T-\varphi+\alpha, \tag{3}$$

$$\sum_{t=\alpha}^{T-\varphi+\alpha-1} \sum_{j=1}^{N_{\alpha+1}} x_{i,\alpha,j,\alpha+1,t} \leq W_{i,\alpha} \times M \qquad i = 1,\ldots,N_\alpha; \quad \alpha = 1,\ldots,\varphi-1, \tag{4}$$

$$\sum_{t=\alpha}^{T-\varphi+\alpha-1} \sum_{i=1}^{N_\alpha} x_{i,\alpha,j,\alpha+1,t} \leq W_{j,\alpha+1} \times M \qquad j = 1,\ldots,N_{\alpha+1}; \quad \alpha = 1,\ldots,\varphi-1, \tag{5}$$

$$h^f_{i,\alpha,t} = h^f_{i,\alpha,t-1} + z_{i,\alpha,t} - \sum_{j=1}^{N_{\alpha+1}} x_{i,\alpha,j,\alpha+1,t}$$

$$i=1,\ldots,N_\alpha;\ \alpha=1,\ldots,\varphi-1;\ t=\alpha,\ldots,T-\varphi+\alpha-1, \tag{6}$$

$$h^f_{i,\varphi,t}=h^f_{i,\varphi,t-1}+z_{i,\varphi,t}-d_tW_{i,\varphi} \qquad i=1,\ldots,N_\varphi; \quad t=\varphi,\ldots,T-1, \tag{7}$$

$$h^r_{i,\alpha,t}=h^r_{i,\alpha,t-1}-z_{i,\alpha,t}+\sum_{j=1}^{N_{\alpha-1}}x_{j,\alpha-1,i,\alpha,t}$$

$$i=1,\ldots,N_\alpha;\quad \alpha=2,\ldots,\varphi;\quad t=\alpha,\ldots,T-\varphi+\alpha, \tag{8}$$

$$h^f_{i,\alpha,0}=0 \qquad i=1,\ldots,N_\alpha;\quad \alpha=1,\ldots,\varphi, \tag{9}$$

$$h^r_{i,\alpha,0}=0 \qquad i=2,\ldots,N_\alpha;\quad \alpha=2,\ldots,\varphi, \tag{10}$$

$$h^f_{i,\alpha,t},h^r_{i,\alpha,t},z_{i,\alpha,t}\geq 0 \qquad i=1,\ldots,N_\alpha;\quad \alpha=1,\ldots,\varphi;\quad t=\alpha,\ldots,T-\varphi+\alpha-1, \tag{11}$$

$$x_{i,\alpha,j,\alpha+1,t}\geq 0 \qquad \forall\ (i,j)\in A;\quad \alpha=1,\ldots,\varphi-1;\quad t=\alpha,\ldots,T-\varphi+\alpha-1, \tag{12}$$

$$Y_{i,\alpha,j,\alpha+1}\geq 0 \qquad \forall\ (i,j)\in A;\quad \alpha=1,\ldots,\varphi-1, \tag{13}$$

$$W_{i,\alpha}\in\{0,1\} \qquad i=1,\ldots,N_\alpha;\quad \alpha=1,\ldots,\varphi. \tag{14}$$

The objective function minimizes the integrated costs of production, inventory holding, transportation and company-selection. Constraints (1) and (2) ensure that the chain of companies chosen from the network includes one company from each echelon[1]. Constraints (3)-(5) ensure that only companies in the virtual organization (selected chain) are allowed to perform production and transportation processes for the product. Constraints (3) contain the capacity limits. Constraints (6)-(8) are the balance equations for the finished goods and raw materials. Note that constraints (6)-(8) are formulated taking into consideration the amount of product produced by a node and not the amount of product delivered by it. This is explained as follows, in order to take advantage of the integration of operations, the formulation considering amount produced, allows to choose the most cost efficient place in

[1]The formulation of the problem can easily be extended to allow the selection of more than one company from any echelon in the network.

which material can be stored if necessary, either in the supplier's location (as finished goods) or in the succeeding location (as raw materials). If the formulation of constraints (6)-(8) considers the amount of product delivered, this flexibility is lost since the supplier will choose to send the material without regards of the production capability or the holding cost of its succeeding location. Constraints (9) and (10) provide the initial conditions of inventory at the nodes. Constraints (11)-(13) indicate that h^f_{it}, h^r_{it}, z_{it}, x_{ijt} and Y_{ij} are non-negative variables, and finally constraint (14) denotes that W_i are binary variables.

Note that the time horizon does not start in period 1 and does not finish in period T for all echelons. This is explained as follows, a node in a given echelon cannot start production until it has received product from some node in its preceding echelon. Given that the production plus transportation time between all echelons is assumed to be one period, the first productive period for all nodes in a given echelon will equal that echelon's position in the sequence of operations, i.e. the echelon's number. Similarly, to ensure that the last demand is met at time T, the last active period for echelon α is $T - \varphi + \alpha$. Also note that if final distances to the customer on the demand side are important, an additional echelon at the final stage with relevant transportation costs can be added. Similar treatment on the supply side can also be made.

4 Path Relaxation Solution Approach

As mentioned earlier, the complexity of the problem lies in the integration of three hard problems: the fixed charge problem for the selection of the nodes, the capacitated multi-stage production planning problem at the selected nodes, and the transportation problem constrained by the previous two problems. Small dimensioned problems of around 4 echelons containing 3 nodes each and a time horizon of 4 periods can be solved using a standard mixed integer linear program solver. However, the nature of the problem requires the use of a heuristic approach to solve problems of higher dimensions. In this section, based on key properties for lower bounds, we develop a path relaxation (PR) algorithm which uses linear programming to solve subproblems of much smaller dimension in an iterative manner. The algorithm can be stopped at an acceptable solution time to obtain a heuristic solution, and has the capability of finding the optimal solution under less time pressure.

We propose a heuristic approach based on the representation of each node's production and each arc's transportation operations by an artificial link to replace the arc and whose "distance" represents the total operational costs of the node-arc combination (i.e., holding and production at node and fixed alliance and transportation costs at an arc). This static distance is a surrogate measure for the effectiveness of this partner and channel in the entire supply chain. Once these distances have been obtained for all the arcs, the problem becomes one of finding the shortest path in the network. For the purpose of our PR algorithm, and proof of optimality, we ensure that the surrogate measure is the lower bound on the actual costs. Since these distances are surrogate measure of a node-arc's performance in isolation, there might exist other supply-chains that exhibit a better solution. It would be desirable then to explore a finite number of ranked shortest paths based on the surrogate measure for final evaluation. Each final evaluation based on a restricted linear program version of the MILP presented in Section 3.2 provides us an upper bound of the optimal solution.

To achieve exploration of multiple "shortest" paths, one can utilize the Double-Sweep method (Shier 1979) to obtain an initial list of k-shortest paths in the network.

4.1 Properties

The following properties will be used in the PR approach. We first present a subproblem $(P_1^{i,\alpha})$ for each node i in echelon α. This problem aims at optimizing, for each node, the cost related to production and holding in the node and the transportation from this node to the accessible nodes of the next echelon (if any), assuming that the quantities produced along the periods meet the demand.

$$(P_1^{i,\alpha}) \quad Min \sum_{j=1}^{N_{\alpha+1}} F_{i,\alpha,j,\alpha+1} W_{j,\alpha+1} + \sum_{t=\alpha}^{T-\varphi+\alpha-1} \sum_{j=1}^{N_{\alpha+1}} C_{i,\alpha,j,\alpha+1,t} x_{i,\alpha,j,\alpha+1,t}$$

$$+ \sum_{t=\alpha}^{T-\varphi+\alpha} (Min(H^f_{i,\alpha,t}, H^r_{j,\alpha+1,t}) h^f_{i,\alpha,t} + U_{i,\alpha,t} z_{i,\alpha,t})$$

subject to:

$$\sum_{j=1}^{N_{\alpha+1}} W_{j,\alpha+1} = 1, \tag{15}$$

$$z_{i,\alpha,t} \leq \Phi_{i,\alpha,t} \qquad t = \alpha, \ldots, T - \varphi + \alpha, \tag{16}$$

$$\sum_{t=\alpha}^{k-\varphi+\alpha} x_{i,\alpha,j,\alpha+1,t} \geq W_{j,\alpha+1} \times \sum_{t=\varphi}^{k} d_t \qquad j = 1, \ldots, N_{\alpha+1}, k = \varphi, \ldots, T, \tag{17}$$

$$h^f_{i,\alpha,t} = h^f_{i,\alpha,t-1} + z_{i,\alpha,t} - \sum_{j=1}^{N_{\alpha+1}} x_{i,\alpha,j,\alpha+1,t} \qquad t = \alpha, \ldots, T - \varphi + \alpha - 1, \tag{18}$$

$$h^f_{i,\alpha,0} = 0, \tag{19}$$

$$h^f_{i,\alpha,t}, z_{i,\alpha,t} \geq 0 \qquad t = \alpha, \ldots, T - \varphi + \alpha - 1, \tag{20}$$

$$x_{i,\alpha,j,\alpha+1,t} \geq 0 \qquad j = 1, \ldots, N_{\alpha+1}; \quad t = \alpha, \ldots, T - \varphi + \alpha - 1, \tag{21}$$

$$W_{j,\alpha+1} \geq 0 \qquad j = 1, \ldots, N_{\alpha+1}. \tag{22}$$

The formulation is self explanatory given the previous discussion of (P). Note that it can be shown that sum of products shipped on a selected arc will equal the cumulative demand for the horizon. This result has been used in constraint (17).

Result 1. We consider a node i in echelon α and its output arcs. Solving the problem $(P_1^{i,\alpha})$ provides a lower bound of the cost incurred by the production in node i in echelon α and the transportation to node j in echelon $\alpha + 1$ for all arcs $(i, j) \in A$.

Proof. Problem $(P_1^{i,\alpha})$ is a relaxed form of problem (P) restricted to the part of the system including node i of echelon α and the transportation to a node of the next echelon $\alpha + 1$.

(a) The production of node i in echelon α is neither constrained from materials supplied from the previous stage (if any) nor constrained by delivery to the next stage (as long as total demand is met).

(b) The inventory holding cost is the minimum between the finished goods holding cost of node i in echelon α and the raw material holding cost of node j in echelon $\alpha + 1$.

(c) The model allows to transport product to more than one node j in the next echelon $\alpha + 1$.

As a consequence, the cost obtained from node i to the next echelon is less than or equal to the cost we would obtain for any feasible solution in which i would be connected to a node of the next echelon.

■

Result 2. In problem $(P_1^{i,\alpha})$ for each node i in echelon α replacing $W_{j,\alpha+1}$ by a binary variable provides a valid and tighter lower bound of the cost incurred by the production in node i in echelon α and the transportation to node j in echelon $\alpha+1$ for all arcs $(i,j) \in A$ (Let this problem be called $(P_{1'}^{i,\alpha})$).

Proof. Problem $(P_{1'}^{i,\alpha})$ is also a relaxed form of problem (P) restricted to the part of the system including node i of echelon α and the transportation to a single "cheapest" node of the next echelon $\alpha+1$. Except for condition (b) all previous arguments hold true. Because transportation to multiple nodes is disallowed in $(P_{1'}^{i,\alpha})$ (as is the feature of (P)), it is a tighter formulation than $(P_1^{i,\alpha})$ and could provide an improved bound.

■

The problems $(P_1^{i,\alpha})$ or $(P_{1'}^{i,\alpha})$ have to be solved at every node of every echelon, so the total number of problems to be solved are equal to $|N|$. $(P_1^{i,\alpha})$ is an LP while $(P_{1'}^{i,\alpha})$ is an MILP of relatively small dimension and should be easy to compute using a standard solver. Depending on the dimension of the original problem (P) if we accept to solve a larger number of problems equal to $|A|$, we can develop a subproblem $(P_2^{i,\alpha,j,\alpha+1})$ for each node i and corresponding arc $(i,j) \in A$.

$$(P_2^{i,\alpha,j,\alpha+1}) \quad Min \; F_{i,\alpha,j,\alpha+1} + \sum_{t=\alpha}^{T-\varphi+\alpha-1} C_{i,\alpha,j,\alpha+1,t} x_{i,\alpha,j,\alpha+1,t}$$

$$+ \sum_{t=\alpha}^{T-\varphi+\alpha} (Min(H^f_{i,\alpha,t}, H^r_{j,\alpha+1,t}) h^f_{i,\alpha,t} + U_{i,\alpha,t} z_{i,\alpha,t})$$

subject to:

$$z_{i,\alpha,t} \leq \Phi_{i,\alpha,t} \qquad t = \alpha, \ldots, T-\varphi+\alpha, \tag{23}$$

$$\sum_{t=\alpha}^{k-\varphi+\alpha} x_{i,\alpha,j,\alpha+1,t} \geq \sum_{t=\varphi}^{k} d_t \qquad k = \varphi, \ldots, T, \tag{24}$$

$$h^f_{i,\alpha,t} = h^f_{i,\alpha,t-1} + z_{i,\alpha,t} - x_{i,\alpha,j,\alpha+1,t} \qquad t = \alpha, \ldots, T - \varphi + \alpha - 1, \tag{25}$$

$$h^f_{i,\alpha,0} = 0, \tag{26}$$

$$h^f_{i,\alpha,t}, z_{i,\alpha,t}, x_{i,\alpha,j,\alpha+1,t} \geq 0 \qquad t = \alpha, \ldots, T - \varphi + \alpha - 1, \tag{27}$$

Result 3. Solving the problem $(P_2^{i,\alpha,j,\alpha+1})$ provides a valid lower bound of the cost incurred by the production in node i in echelon α and the transportation to node j in echelon $\alpha+1$ for all arcs $(i,j) \in A$. Also this bound is of better or equal quality than that obtained by solving subproblems $(P_1^{i,\alpha})$ or $(P_{1'}^{i,\alpha})$ which assign the same value to every arc issuing from node i.

Proof. The first part can be seen by similar arguments as previously. The second part can be shown by the fact that $(P_2^{i,\alpha,j,\alpha+1})$ is for each node when one of its outgoing arc is considered separately, while $(P_{1'}^{i,\alpha})$ picks the cheapest outgoing arc, say j^*, when all outgoing arcs are considered simultaneously. $(P_1^{i,\alpha})$ then assigns the same distance to all the outgoing arcs. For the selected node j^*, $(P_2^{i,\alpha,j^*,\alpha+1})$ provides the same value as $(P_{1'}^{i,\alpha})$. For another node j' of echelon $\alpha+1$, $(i,j') \in A$ and $j' \neq j^*$, the integrated costs of $(P_2^{i,\alpha,j^*,\alpha+1})$ will be at least as much as that of $(P_{1'}^{i,\alpha})$.

■

4.2 PR Algorithm

Algorithm 1

Step 1: Surrogate lengths of arcs.

For each node i in each echelon α, $i = 1, \ldots, N_\alpha$; $\alpha = 1, \ldots, \varphi$:

1. Solve problem $(P_1^{i,\alpha})$ (or $(P_{1'}^{i,\alpha})$).
2. For each outgoing arc of i $(i,j) \in A$ assign the length $\ell_{i,j}$ equal to the criterion provided by the optimal solution of problem $(P_1^{i,\alpha})$ (or $(P_{1'}^{i,\alpha})$).

Step 2: Search for a near-optimal solution.

1. Set iteration $k = 1$, lowerbound $LB = 0$, upperbound $UB = \infty$, and initialize best path $\mathcal{L}^* = \phi$.

2. Compute the k^{th} shortest path using the surrogate lengths of the arcs.

   If no such path exists, exit.

   Otherwise, let $\mathcal{L}_k$ be the path and $\mathcal{V}_k$ the value of the criterion. $\mathcal{V}_k$ is a lowerbound of the optimal cost of the original problem (P) with any previously explored path eliminated. If $\mathcal{V}_k > LB$, set $LB = \mathcal{V}_k$. Note the label LB may not represent the lower bound of the overall problem (P), but of the remaining unexplored problem.

3. Starting from the nodes in $\mathcal{L}_k$ and introducing the corresponding values of binary variables $W_{i,\alpha}$) in problem (P), solve the restricted version of the problem (P) called (P_k^R).

   If (P_k^R) has an optimal solution, the value of its criterion $\mathcal{C}_k$ is an upperbound of the optimal cost of the original problem (P). Note that (P_k^R) is a linear program. If $\mathcal{C}_k < UB$, set $UB = \mathcal{C}_k$ and $\mathcal{L}^* = \mathcal{L}_k$.

   If $UB \leq LB$, exit with optimal $\mathcal{L}^*$ as the supply-chain.

4. If user wishes to stop, exit.

   Otherwise $k = k + 1$ and return to Step 2.2.

Algorithm 2

Algorithm 2 is similar to the previous PR algorithm, except for Step 1 that becomes:

Step 1: Surrogate lengths of arcs.

For each node i in each echelon α, $i = 1, \ldots, N_\alpha$; $\alpha = 1, \ldots, \varphi - 1$ and its outgoing arc $(i, j) \in A$ (note, node j is in echelon $\alpha + 1$):

1. Solve problem $(P_2^{i,\alpha,j,\alpha+1})$.

2. For an arc of (emanating from) node i, $(i, j) \in A$, assign the length $\ell_{i,j}$ equal to the criterion provided by the optimal solution of problem $(P_2^{i,\alpha,j,\alpha+1})$.

Result 4. Algorithm 1 (or 2) leads to an optimal solution if the user does not interrupt it at step 4.

Proof. The algorithm stops when the lower bound LB of the k^{th} path exceeds the lowest upper bound obtained so far, and this lowest upper bound is the total cost corresponding to a feasible supply chain $\mathcal{L}^*$. Furthermore, all the paths that have not been explored so far have a lower bound that is greater than or equal to LB. Thus, all the solutions that have not been considered so far will lead to a solution no better than the feasible supply chain $\mathcal{L}^*$.

4.3 Simplification for time invariant costs

When the cost terms of unit production cost and unit holding costs at each node, and unit transportation cost on each arc are the same for all time periods of the planning horizon, certain simplifications in the lower bound calculations emerge. In particular for problem $(P_2^{i,\alpha,j,\alpha+1})$, the second term in the objective function $\sum_{t=\alpha}^{T-\varphi+\alpha-1} C_{i,\alpha,j,\alpha+1,t}x_{i,\alpha,j,\alpha+1,t}$ simplifies to a constant $C_{i,\alpha,j,\alpha+1}\sum_{t=\varphi}^{T} d_t$. In addition, it can be shown that this problem will have no holding so $\sum_{t=\alpha}^{T-\varphi+\alpha} H_{i,\alpha}^f h_{i,\alpha,t}^f = 0$, and the final term in the objective function becomes a constant $U_{i,\alpha}\sum_{t=\alpha}^{T-\varphi+\alpha} z_{i,\alpha,t} = U_{i,\alpha}\sum_{t=\varphi}^{T} d_t$. All these can be evaluated without the need of a linear programming solver.

5 Numerical Example and Computational Tests

5.1 An Illustrative Example

We consider the problem of 4-echelon network, with 5 nodes in each. The jth node of ith echelon is numbered as $(i-1)*5+j$. Thus, the nodes of the last echelon are 16,17,18,19 and 20. Demand per period are 300,360,340 and 300. Each node of echelon $i(i=1,2,3)$ is connected with each node of echelon $i+1$.The fixed cost and the transportation cost are given in table 1,2 and 3. The unit processing costs, raw material holding cost and the finished goods holding cost for each node are displayed in table 4. We assumed that transportation cost is same for all periods and zero transportation cost for echelon 4 to final demand producing unit.

Table 1: Fixed cost and transportation cost: Echelon 1 to 2

Nodes	Node6		Node7		Node8		Node9		Node10	
	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.
Node1	1400	10.5	1500	12.5	1600	11.5	1300	14.0	1500	12.0
Node2	1300	11.0	1500	12.8	1700	12.5	1750	11.0	1900	12.0
Node3	1100	11.0	2000	14.0	1680	12.0	1300	14.0	2000	14.0
Node4	1200	12.0	1300	12.0	1450	12.6	1650	14.0	1500	18.0
Node5	1700	10.5	1600	13.0	1570	10.6	1700	14.0	2000	14.0

Table 2: Fixed cost and transportation cost: Echelon 2 to 3

Nodes	Node11		Node12		Node13		Node14		Node15	
	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.
Node6	1200	14.2	1700	15.0	1600	12.0	1400	13.5	1780	8.0
Node7	1800	15.6	1400	12.5	1800	14.0	1750	16.5	1980	11.0
Node8	1720	19.8	1900	17.6	1900	14.5	1980	12.5	1700	12.5
Node9	1820	12.5	1600	18.1	2200	12.5	1270	12.0	1900	16.5
Node10	1880	15.4	1200	12.5	1500	16.0	1180	17.0	1800	16.0

Table 3: Fixed cost and transportation cost: Echelon 3 to 4

Nodes	Node16		Node17		Node18		Node19		Node20	
	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.	Fixed	Transp.
Node11	2700	12.5	1800	16.0	1900	16.5	1100	9.8	2500	16.0
Node12	1800	17.0	1200	13.0	1400	12.0	1900	10.5	1000	11.0
Node13	1100	10.5	1400	11.0	1900	13.5	1600	18.0	1050	12.0
Node14	1350	14.5	1600	9.5	1800	19.0	1780	14.0	1850	14.0
Node15	1600	17.5	1100	8.5	1400	10.5	1300	14.0	1250	10.0

We first solve the problem $(P_2^{i,\alpha,j,\alpha+1})$ for each arc. We consider the 15 shortest paths for the example and evaluted each ones feasibility. The lower bound and the upper bound of the paths are presented in table 5. We can see that the lower bound of shortest path eleven is greater than the upperbound of path seven. Thus, we conclude that the upperbound of the seventh shortest path is an optimal cost and the corresponding shortest path is an optimal selection.

Table 4: Parameter Values for the Numerical Example

Parameters	Capacity				Processing cost/unit				Raw Mtls holding cost (per unit)	Finished goods holding cost (per unit)
	1	2	3	4	1	2	3	4		
Node1	333	312	329	321	33	32	35	42	0	10.25
Node2	340	345	300	360	38	37	39	39	0	8.25
Node3	330	330	330	330	37	39	37	37	0	8.70
Node4	400	400	380	380	34	36	35	40	0	9.0
Node5	370	360	360	360	38	36	36	36	0	8.50
Node6	340	340	340	340	55	56	58	59	6.15	12.25
Node7	340	333	330	410	52	54	55	54	7.11	14.75
Node8	360	340	360	410	54	56	55	54	7.89	12.75
Node9	340	320	330	390	51	54	52	51	7.55	12.50
Node10	345	331	330	330	51	53	52	51	7.65	12.0
Node11	370	360	330	300	49	49	52	51	8.85	14.70
Node12	370	360	330	300	49	49	50	51	8.80	14.80
Node13	350	350	350	350	49	49	51	50	8.60	14.50
Node14	360	350	350	350	53	53	52	53	8.40	14.0
Node15	360	360	360	360	55	52	54	52	7.5	14.35
Node16	390	370	340	350	48	51	52	51	6.87	14.55
Node17	370	360	340	390	49	52	52	51	7.59	15.45
Node18	350	370	330	400	49	49	52	51	7.65	15.55
Node19	350	340	350	300	49	49	52	51	7.65	15.0
Node20	370	380	350	250	49	49	49	49	7.8	15.0

Table 5: Result

S. No.	Shortest path	Lower bound	Upper bound
1	4-7-12-20	294007	Infeasible
2	4-7-12-19	295787	297409
3	5-10-12-20	295840	Infeasible
4	5-6-15-20	296040	Infeasible
5	4-6-15-20	296190	Infeasible
6	4-7-15-20	296257	Infeasible
7	5-6-15-17	296430	296910
8	4-6-15-17	296580	297940
9	4-7-15-17	296647	298115
10	5-7-12-20	296907	Infeasible
11	4-7-12-18	297147	298761

5.2 Computational Study

To test the hypothesis, we randomly generated 50 examples for 5 echelons in which each echelon contains 2 to 6 nodes. For each example we considered an horizon of four time periods and computed 50 shortest paths. All the examples are generated in such a way that it contains at least one feasible solution. The examples are solved using algorithm 2. The results are summerised in table 6.

Table 6: Computational study

Shortest path $n1 - n2$	Number of examples in which an optimal solution is found after exploring between $n1$ to $n2$ shortest paths
1 - 5	7
6 - 10	13
11 - 20	22
21 - 30	3
31 - 40	1
41 - 50	2

Out of the 50 problems, the first shortest path gives the optimal solution in 29 cases, the second shortest path gives the optimal solution in 9 cases and third shortest path gives the optimal solution in 3 cases. In more than 90 percent cases, the optimal solution lies within the first 10 shortest paths. But, in some of the cases, we find it after exploring more than 30 shortest paths. We didn't find the optimal solution for two problems even after exploring 50 shortest paths. It is possible, for these two examples, that optimal solution may exist within 50 shortest paths and we need to explore more than 50 shortest paths to find a lower bound that exceeds the upper bound obtained from the 50 shortest paths. The decision of computing the number of shortest paths depends on the size of the problem and the cost structure.

6 Conclusion

In this paper we have formulated the problem faced at the strategic level of supply-chain design for a new market opportunity (product or service) from a pool of qualified partners. The product or service is assumed to be generated in a series of stages called echelons. At each echelon we have one or more facilities that can process the correspondent operation. Based on the prevailing workload conditions and the ensuing remaining capacity to be allocated to this new opportunity, the cheapest or the nearest partner might not be the optimal choice. For a projected deterministic forecast over a horizon, the coordinator of the organizational web is interested in extracting a supply-chain of optimal set of partners, one at each echelon, who are capable of meeting the forecast without backlog at minimum cost. The cost is the integrated cost of production and holding at each selected partner and the fixed alliance formation and incremental transportation costs from echelon to echelon. Due to the complexity of the problem we develop a solution approach based on key properties for lower bound nodal plus arc costs to obtain a relaxed shortest path. The procedure evaluates this path for the original problem and iteratively generates the next shortest path in sequence. The procedure is termed as path relaxation and at each iteration a lower and upper bound are obtained. Thus the procedure is capable of generating fast heuristic solutions as well as provable optimal solutions within reasonable time. Computational experiment conducted on randomly generated test problems shows the efficacy of the proposed solution approach. A major advantage of the proposed approach is that it can well deal with large networks in which the interdependence between echelons is considered. As future search, we propose to investigate the stability of static solutions to demand uncertainty.

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